Stochastic Optimization
IDA PhD course 2011ht

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2. Lecture: Uncertainties in objective
06. October 2011

1. Randomness occurs in the objective function
   - Expected value objective function
   - Probability of shortfall
   - Minimize Variance
   - Value at risk

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2. One more SP example
   - Machine Scheduling

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   - Expected value objective function
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3. A bit of History

Outline

Deterministic Opt. Model → Stochastic Programming Model

$$\min_{x \in X} \quad f(x)$$
$$\text{s.t.} \quad g(x) \leq 0$$

Deterministic Opt. Model → Stochastic Programming Model

$$\begin{aligned} \min_{x \in X} \quad & f(x) \\ \text{s.t.} \quad & g(x) \leq 0 \end{aligned} \quad \rightarrow \quad \begin{aligned} \min_{x \in X} \quad & f(x, \chi) \\ \text{s.t.} \quad & g(x, \chi) \leq 0 \end{aligned}$$

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$\chi \in \Omega \subseteq \mathbb{R}^s$: random vector

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$\chi \in \Omega \subseteq \mathbb{R}^s$: vector with random variable as entries

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Minimize an expected value function

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Examples

- Expected cost / Expected gain

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Examples

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Examples

- Expected cost / Expected gain
- Expected machine working time
- Expected transportation time
- Expected customer waiting times
- Expected damage on target

Advantages

- Good result "on average"

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- Objective function can often be reformulated deterministically

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Theorem (Jensen, 1906)

Let f be a convex function and X a random variable. Then

$$\mathbb{E}[f(X)] \geq f(\mathbb{E}[X])$$

Advantages

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Disadvantages

- We might encounter very "bad cases" ("Risk")

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Disadvantages

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- Expectation can only be computed as multidimensional integral

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Linear Programming Problem

Stochastic Programming Problem

Linear Programming Problem

Stochastic Programming Problem

$$\min_{\substack{x \in \mathbb{R}^n \\ x \geq 0}} \quad \mathbb{E}\left[c(\chi)^T x\right]$$

$$\text{s.t.} \quad Ax \leq b$$

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Linear Programming Problem

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$\mu \in \mathbb{R}^n$: (deterministic) vector of means

Discrete Finite Distribution

Stochastic Programming Problem

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$\chi^1, \ldots, \chi^K \in \mathbb{R}^s$: scenarios

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General problem with discrete finite distributions

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Exponential number of scenarios

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Assume:

- Discretely distributed random variables

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General problem with discrete finite distributions

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Assume:

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- n decision variables
- n random variables
- 2 possible outcomes for each random variable (e.g. Bernoulli distribution)

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Independent random variables $\Rightarrow 2^n$ scenarios

Discrete Finite Distribution

Stochastic Programming Problem

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Outline

Minimize probability of shortfall

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- Investment strategies
- Project cost management ($T = 0$)

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- Investment strategies
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Probability of "Target" achievement

Advantages

- If probability of shortfall too high actions can be taken.

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Disadvantages

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Advantages

- If probability of shortfall too high actions can be taken.

Disadvantages

- We might still encounter very "bad cases"
- No influence on average cost

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Discrete Finite Distribution

Stochastic Programming Problem

Discrete Finite Distribution

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Reformulate Problem Deterministically

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Basic idea:

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Basic idea:

- "Choose" scenarios with shortfall

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Basic idea:

- "Choose" scenarios with shortfall
- Probability that one of these arises minimized

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Realization:

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Basic idea:

- "Choose" scenarios with shortfall
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Realization:

- Introduce one binary decision variable z^k per scenario

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Reformulate Problem Deterministically

Basic idea:

- "Choose" scenarios with shortfall
- Probability that one of these arises minimized

Realization:

- Introduce one binary decision variable z^k per scenario
- $z^k = 1$: shortfall in scenario k

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Discrete Finite Distribution II

Reformulate Problem Deterministically

Basic idea:

- "Choose" scenarios with shortfall
- Probability that one of these arises minimized

Realization:

- Introduce one binary decision variable z^k per scenario
- $z^k = 1$: shortfall in scenario k

Discrete Finite Distribution II

Reformulate Problem Deterministically

Basic idea:

- "Choose" scenarios with shortfall
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Deterministically reformulated problem

Discrete Finite Distribution II

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Deterministically reformulated problem

$$\begin{aligned} \min \quad & \\ \text{s.t.} \quad & g(x) \leq 0 \\ & x \in X \end{aligned}$$

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Deterministically reformulated problem

$$\begin{aligned} \min \quad & \\ \text{s.t.} \quad & g(x) \leq 0 \\ & x \in X, \quad z^k \in \{0,1\} \quad \forall k = 1, \ldots, K \end{aligned}$$

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Discrete Finite Distribution II

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Deterministically reformulated problem

$$\min$$

$$\begin{aligned} \text{s.t.} \quad & g(x) \leq 0 \\ & f(x, \chi^k) \leq T + Mz^k \\ & x \in X, \quad z^k \in \{0,1\} \quad \forall k = 1, \ldots, K \end{aligned}$$

M: some "big" constant

Discrete Finite Distribution II

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Discrete Finite Distribution III

Deterministically reformulated problem

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Problems

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Problems

- Numerical instability due to big M possible

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Deterministically reformulated problem

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M: some "big" constant

Problems

- Numerical instability due to big M possible
- K additional constraints

Discrete Finite Distribution III

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$$

M: some "big" constant

Problems

- Numerical instability due to big M possible
- K additional constraints
- K additional binary decision variables
- Deterministic reformulation hard

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f linear / Normal Distribution

Stochastic Programming Problem

f linear / Normal Distribution

Stochastic Programming Problem

$$\min_{x \in X} \quad \mathbb{P}\{\chi^T x > T\}$$
$$\text{s.t.} \quad g(x) \leq 0$$

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$\chi \sim \mathcal{N}(\mu, \Sigma)$
Σ: Covariance Matrix of χ

Deterministically reformulated problem

$$\max_{x \in X} \quad \frac{T - \mu^T x}{\sqrt{x^T \Sigma x}}$$
$$\text{s.t.} \quad g(x) \leq 0$$

$x^* \neq 0$

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Outline

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Minimize variance

Minimize variance

$$\min_{x \in X} \quad Var\left[f(x,\chi)\right]$$
$$\text{s.t.} \quad g(x) \leq 0$$

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Advantages:

Minimize variance

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Advantages:

- Outcome more concentrated around mean

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Minimize variance ?

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Advantages:

- Outcome more concentrated around mean
- Possibility to reduce risk

Disadvantages:

- Makes not much sense without benchmark for expected costs

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Simple Mean-Variance Models

Minimize convex combination of variance and expectation

Simple Mean-Variance Models

Minimize convex combination of variance and expectation

$$\min_{x \in X} \quad \lambda Var\left[f(x,\chi)\right] + (1-\lambda)\mathbb{E}\left[f(x,\chi)\right]$$
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Simple Mean-Variance Models

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Simple Mean-Variance Models

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Minimize variance with expectation threshold

Simple Mean-Variance Models

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Problems when variance in objective

- Loss of linearity

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- Hardness of problem (e.g. quadratic objective)

Problems when variance in objective

- Loss of linearity
- Loss of convexity
- Hardness of problem (e.g. quadratic objective)
- Compute variance / Evaluate objective function

Outline

1. Randomness occurs in the objective function
   - Expected value objective function
   - Probability of shortfall
   - Minimize Variance
   - Value at risk
2. One more SP example
   - Machine Scheduling
3. A bit of History

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Question

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Question

What is the probability that my total loss during a fixed time interval does not exceed a certain limit?

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What is the probability that my total loss during a fixed time interval does not exceed a certain limit?

Examples

Question

What is the probability that my total loss during a fixed time interval does not exceed a certain limit?

Examples

- What is the probability that my stock portfolio will fall in value by more than $ 100 million in one week?

Question

What is the probability that my total loss during a fixed time interval does not exceed a certain limit?

Examples

- What is the probability that my stock portfolio will fall in value by more than $ 100 million in one week?
- If I invest $ 1 million today, how much can I loose till tomorrow?

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Definition (Value-at-Risk)

X: random variable describing the loss over time horizon T

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Φ_X: Cumulative distribution function of X

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Value at risk over time horizon T at confidence level α:

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Value at risk over time horizon T at confidence level α:

$$VAR_\alpha(X) = \inf\{c | \Phi_X(c) \geq \alpha\}$$

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Interpretation (Philippe Jorion)

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Definition (Value-at-Risk)

X: random variable describing the loss over time horizon T
Φ_X: Cumulative distribution function of X

Value at risk over time horizon T at confidence level α:

$$VAR_\alpha(X) = \inf\{c|\Phi_X(c) \geq \alpha\}$$

Interpretation (Philippe Jorion)

"Value at Risk measures the worst expected loss over a given horizon under normal market conditions at a given level of confidence."

Value at Risk in Stochastic Programming

- Risk measure

Value at Risk in Stochastic Programming

- Risk measure
- Objective: Minimize value at risk

Value at Risk in Stochastic Programming

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Critics

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Critics

- Lack of subadditivity

Value at Risk in Stochastic Programming

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- Difficult to compute from scenarios

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Alternatives

- Conditional value at risk

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Value at Risk in Stochastic Programming

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- Conditional value at risk
- Tail value at risk

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Outline

Outline

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Deterministic Problem

(Possible) Parameters

Deterministic Problem

(Possible) Parameters

- # of (different) machines / parts

Deterministic Problem

(Possible) Parameters

- # of (different) machines / parts
- Processing times

Deterministic Problem

(Possible) Parameters

- # of (different) machines / parts
- Processing times
- # of jobs to be completed

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Deterministic Problem

(Possible) Parameters

- # of (different) machines / parts
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Deterministic Problem

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- # of (different) machines / parts
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- Due dates

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(Possible) Objectives

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Deterministic Problem

(Possible) Parameters

- # of (different) machines / parts
- Processing times
- # of jobs to be completed
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- Precedence relations

(Possible) Objectives

- Minimize total completition time

Deterministic Problem

(Possible) Parameters

- # of (different) machines / parts
- Processing times
- # of jobs to be completed
- # of employees available
- Due dates
- Precedence relations

(Possible) Objectives

- Minimize total completition time
- Maximize # of completed jobs

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Deterministic Problem

(Possible) Parameters

- # of (different) machines / parts
- Processing times
- # of jobs to be completed
- # of employees available
- Due dates
- Precedence relations

(Possible) Objectives

- Minimize total completition time
- Maximize # of completed jobs
- Minimize maximum/sum of tardyness

Deterministic Problem

(Possible) Parameters

- # of (different) machines / parts
- Processing times
- # of jobs to be completed
- # of employees available
- Due dates
- Precedence relations

(Possible) Objectives

- Minimize total completion time
- Maximize # of completed jobs
- Minimize maximum/sum of tardyness
- Minimize idle times

Stochastic Problem

(Possible) Uncertain Parameters

Stochastic Problem

(Possible) Uncertain Parameters

- # of (different) available machines

Stochastic Problem

(Possible) Uncertain Parameters

- # of (different) available machines $\leftarrow$ break downs

Stochastic Problem

(Possible) Uncertain Parameters

- # of (different) available machines $\leftarrow$ break downs
- # of (different) parts

Stochastic Problem

(Possible) Uncertain Parameters

- # of (different) available machines $\leftarrow$ break downs
- # of (different) parts $\leftarrow$ costumization

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Stochastic Problem

(Possible) Uncertain Parameters

- # of (different) available machines $\leftarrow$ break downs
- # of (different) parts $\leftarrow$ costumization
- Processing times

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Stochastic Problem

(Possible) Uncertain Parameters

- # of (different) available machines $\leftarrow$ break downs
- # of (different) parts $\leftarrow$ costumization
- Processing times $\leftarrow$ manual operations

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Stochastic Problem

(Possible) Uncertain Parameters

- # of (different) available machines ← break downs
- # of (different) parts ← costumization
- Processing times ← manual operations
- # of jobs to be completed

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Stochastic Problem

(Possible) Uncertain Parameters

- # of (different) available machines $\leftarrow$ break downs
- # of (different) parts $\leftarrow$ costumization
- Processing times $\leftarrow$ manual operations
- # of jobs to be completed $\leftarrow$ demand

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Stochastic Problem

(Possible) Uncertain Parameters

- # of (different) available machines ← break downs
- # of (different) parts ← costumization
- Processing times ← manual operations
- # of jobs to be completed ← demand
- # of employees available

Stochastic Problem

(Possible) Uncertain Parameters

- # of (different) available machines ← break downs
- # of (different) parts ← costumization
- Processing times ← manual operations
- # of jobs to be completed ← demand
- # of employees available ← sickness, vacations

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- # of (different) parts ← costumization
- Processing times ← manual operations
- # of jobs to be completed ← demand
- # of employees available ← sickness, vacations
- Due dates

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- # of (different) parts $\leftarrow$ costumization
- Processing times $\leftarrow$ manual operations
- # of jobs to be completed $\leftarrow$ demand
- # of employees available $\leftarrow$ sickness, vacations
- Due dates $\leftarrow$ uncertainty in processing times
- Precedence relations

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- # of employees available ← sickness, vacations
- Due dates ← uncertainty in processing times
- ~~Precedence relations~~

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Stochastic Problem

(Possible) Objective

Stochastic Problem

(Possible) Objective

- minimize expected...

Stochastic Problem

(Possible) Objective

- minimize expected... total processing time

Stochastic Problem

(Possible) Objective

- minimize expected... total processing time
- Given # of jobbs, maximize probability that...

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Stochastic Problem

(Possible) Objective

- minimize expected... total processing time
- Given # of jobbs, maximize probability that... processing "in time"

Stochastic Problem

(Possible) Objective

- minimize expected... total processing time
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Stochastic Settings

Stochastic Problem

(Possible) Objective

- minimize expected... total processing time
- Given # of jobbs, maximize probability that... processing "in time"

Stochastic Settings

- Single stage decision

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Stochastic Problem

(Possible) Objective

- minimize expected... total processing time
- Given # of jobbs, maximize probability that... processing "in time"

Stochastic Settings

- Single stage decision
- Multi-Stage decision

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Stochastic Problem

(Possible) Objective

- minimize expected... total processing time
- Given # of jobbs, maximize probability that... processing "in time"

Stochastic Settings

- Single stage decision
- Multi-Stage decision $\leftarrow$ Discretization of processing time

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Stochastic Problem

(Possible) Objective

- minimize expected... total processing time
- Given # of jobbs, maximize probability that... processing "in time"

Stochastic Settings

- Single stage decision
- Multi-Stage decision ← Discretization of processing time
- Online Programming

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Stochastic Problem

(Possible) Objective

- minimize expected... total processing time
- Given # of jobbs, maximize probability that... processing "in time"

Stochastic Settings

- Single stage decision
- Multi-Stage decision $\leftarrow$ Discretization of processing time
- Online Programming $\leftarrow$ New information arrives over time

Outline

The beginning

George Dantzig
Linear programming under uncertainty. (1955)
Management Science 1:197–206

The beginning

George Dantzig
Linear programming under uncertainty. (1955)
Management Science 1:197–206

- Two-Stage and Simple recourse problems

The beginning

George Dantzig
Linear programming under uncertainty. (1955)
Management Science 1:197–206

- Two-Stage and Simple recourse problems
- Finite number of scenarios

The beginning

George Dantzig
Linear programming under uncertainty. (1955)
Management Science 1:197–206

- Two-Stage and Simple recourse problems
- Finite number of scenarios
- Deterministic Reformualtion

The beginning

George Dantzig
Linear programming under uncertainty. (1955)
Management Science 1:197–206

- Two-Stage and Simple recourse problems
- Finite number of scenarios
- Deterministic Reformualtion
- No use of special structure

Richard Van Slyke and Roger J-B. Wets
L-shaped linear programs with applications to optimal control and stochastic programming. (1969)
MSIAM Journal on Applied Mathematics 17(4):638–663, 1969

Richard Van Slyke and Roger J-B. Wets
L-shaped linear programs with applications to optimal control and stochastic programming. (1969)
MSIAM Journal on Applied Mathematics 17(4):638–663, 1969

- Solution method that makes use of special problem structures

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L-shaped linear programs with applications to optimal control and stochastic programming. (1969)
MSIAM Journal on Applied Mathematics 17(4):638–663, 1969

- Solution method that makes use of special problem structures
- Reduced computing time

András Prékopa
On probabilistic constrained programming. (1970)
Proceedings of the Princeton Symposium on Mathematical Programming 113–1383

András Prékopa
A class of stochastic programming decision problems. (1972)
Mathematische Operationsforschung und Statistik 3(5):349–354

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- Main contributions to understanding of chance-constraint programming

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- Main contributions to understanding of chance-constraint programming
- Convex cases

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- Convex cases
- Joint constraints

Maarten H. van der Vlerk
Stochastic Programming with Integer Recourse. (1995)
PhD thesis, University of Groningen, The Netherlands

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Stochastic Programming with Integer Recourse. (1995)
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- Main contributions to understanding of Integer Programming with Recourse

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Stochastic Programming with Integer Recourse. (1995)
PhD thesis, University of Groningen, The Netherlands

- Main contributions to understanding of Integer Programming with Recourse
- with Leen Stougie, Rüdiger Schultz

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Alexander Shapiro and Tito Homem-de-Mello
A simulation-based approach to two-stage stochastic programming with recourse. (1998)
Mathematical Programming 81(3):301-325

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- Stochastic Programming via Monte Carlo Sampling: Sample Average Approach

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- Stochastic Programming via Monte Carlo Sampling: Sample Average Approach
- Much work on convergence properties

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A simulation-based approach to two-stage stochastic programming with recourse. (1998)
Mathematical Programming 81(3):301-325

- Stochastic Programming via Monte Carlo Sampling: Sample Average Approach
- Much work on convergence properties
- Realization: Often good approximations possible with "relatively" few samples

Next lecture

- Chance-Constrained Programming and related problems

Next lecture

- Chance-Constrained Programming and related problems
- (Simple Recourse Problems)

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QUESTIONS?

What about ~~next week~~ in 2 weeks?